Applied economic networks - IV

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Today's class

- Strategic network formation.
- Mushegh's presentation: *The law of the few*.
- Robin's presentation: *The network structure of international trade*.

Takeaway from random networks

- **Erdos-Renyi model**: Pure randomness, nice properties.
- **Barabasi-Albert model**: Preferential attachment, explains effectively power-law degree distributions.
- **Watts-Strogatz model**: Rewiring from a circle, range of rewiring probability that generates low diameter and high clustering.
- **Jackson-Rogers model**: Randomness and preferential attachment, flexible model that can be adapted to fit lot of different data.

A general consideration

Random network models

- **Positive analysis**: Random models are great to explain a specific configuration
- **Normative analysis**: good to analyze big shocks to the overall functioning of the system.

Strategic models

- **Positive analysis**: a primary goal is to identify a strategy that matches observed behavior
- **Normative analysis**: if the theory is good, then it is useful to formulate predictions.

Arguments for strategic models

- Why?
- What about efficiency?
- Empirics?

How do we build a model of network formation?

1. Costs and benefits associated to a network configuration.
2. How agents choose links.

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Results

- What network configurations are likely to form?
- Analyze social efficiency: is there a tension between stability and efficiency?

Plenty of choices

Incentive to form and delete links:

- is consensus needed?
- is the process dynamic or static?
- how sophisticated are the agents?
- what do agents know when making a decision?
- do errors happen?
- can agents compensate each other?
- can agents adjust the intensity of a link?

Jackson Wolinsky (1996)

Among the first economic papers on networks.
Practically, it has initiated the field of **Social and Economic Networks**.
Published in 1996, on the *Journal of Economic Theory*.

The main idea

Agents obtain payoff $u(g)$ that depend on the network structure g.
Two models with undirected network formation.

The connections model

- Agents obtain a benefit from direct and indirect connections.
- $\delta \in [0, 1]$ is the benefit between an agent i and an agent j.
- Links are costly: $c_{ij} \geq 0$ is the cost to i of a link to j.
- $\ell(i, j)$ is the shortest path between i and j.

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Payoff

$$u_i(g) = \sum_j \delta^{\ell(i,j)} - \sum_{j \in N_i(g)} c_{ij}$$

The main questions

- Which networks will the agents form?
- Which networks are the best for the whole society?

Main solution concepts

- **Non-cooperative approach**: Individuals act individually.
  $\Longrightarrow$ Nash equilibrium

- **Cooperative approach**: Individuals operate in groups.
  $\Longrightarrow$ Pairwise Stability (this paper), Pairwise Nash stability,...

An example of Nash equilibrium

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- Nash equilibrium: no incentives to deviate.

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Pairwise stability

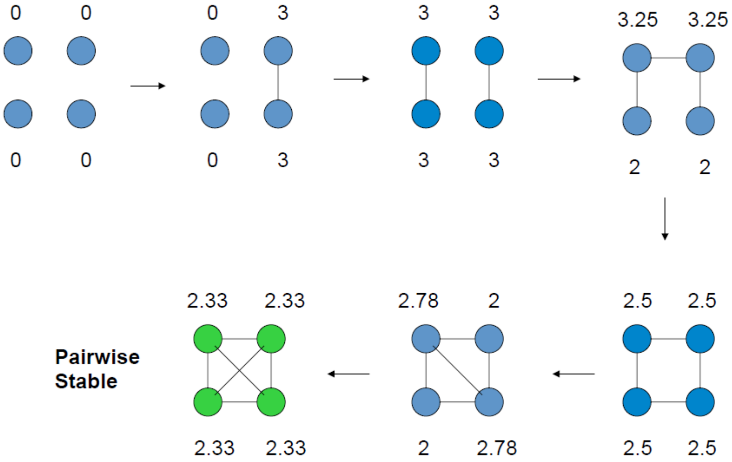
- $$u_i(g) \geq u_i(g - ij) \quad \text{for} \quad ij \in g$$

  No agent gains from deleting a link.

- $$u_i(g + ij) \geq u_i(g)$$
  $$u_j(g + ij) \geq u_j(g)$$

  No two agents both gain from adding a link (at least one strictly)

An example of pairwise stability



Efficiency

Pareto efficiency

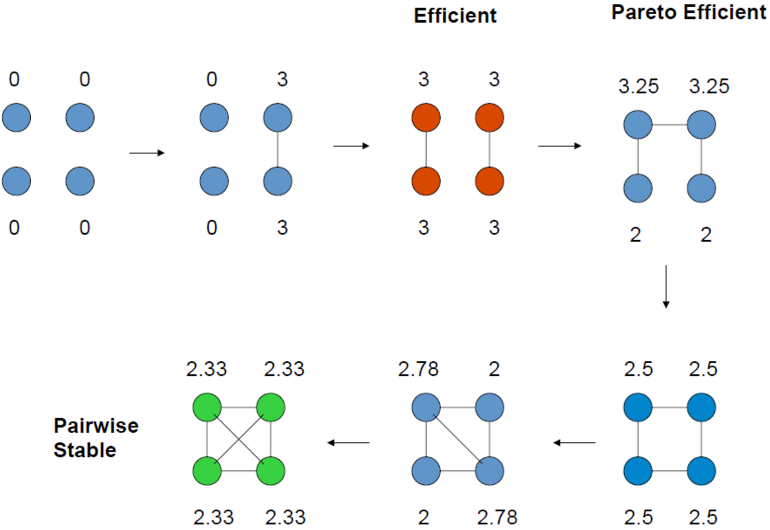
A network g is Pareto efficient if there does not exist a network g' such that:

$$u_i(g') \geq u_i(g) \quad \text{for all} \quad i.$$

Efficiency

A network g is efficient if it maximizes $\sum u_i(g)$.

Back to our example



A generalization of the "connections model"

A more general version, called the distance based utility model (Bloch Jackson, 2005)

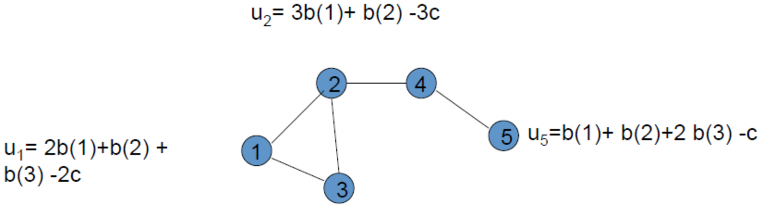
- Let b be a decreasing function
- Links are costly: $c \geq 0$ is the cost of a link.
- $\ell(i, j)$ is the distance between i and j.

Payoff

$$u_i(g) = \sum_j b(\ell(i, j)) - d_i(g)c$$

Intuition

- The benefit from a friend is $b(1)$
- The benefit from a friend of a friend is $b(2) < b(1)$,...
- Cost $c > 0$



Efficient networks

- **Low cost**: $c < b(1) - b(2) \implies$ the complete network is uniquely efficient.

- **Medium cost**: $b(1) - b(2) < c < b(1) + (n-2)b(2)/2 \implies$ star networks are uniquely efficient.

- **High cost**: $c > b(1) + (n-2)b(2)/2 \implies$ empty network is uniquely efficient.

Sketch of the proof

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The value of a network with k players and m links is at most:

$$2m[b(1)-c]+[k(k-1)-2m]b(2)$$

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If we take the difference we obtain:

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replaced with a link at distance 3. The star is better.What if more

than one star?

Pairwise stability

- **Low cost**: $c < b(1) - b(2) \implies$ the complete network is pairwise stable.
- **Medium- low cost**: $b(1) - b(2) < c < b(1) \implies$ star networks are pairwise stable, but not uniquely.
- **Medium- high cost**: $b(1) < c < b(1) + (n-2)b(2)/2 \implies$ star networks are not pairwise stable, and the pairwise stable networks are over-connected and with few agents.
- **High cost**: $c > b(1) + (n-2)b(2)/2 \implies$ empty network is uniquely pairwise stable.

Externalitites

- **Positive externalities**: As in the connections model, there is an inefficiency.
- **Negative externalities**: ?

The coauthor model

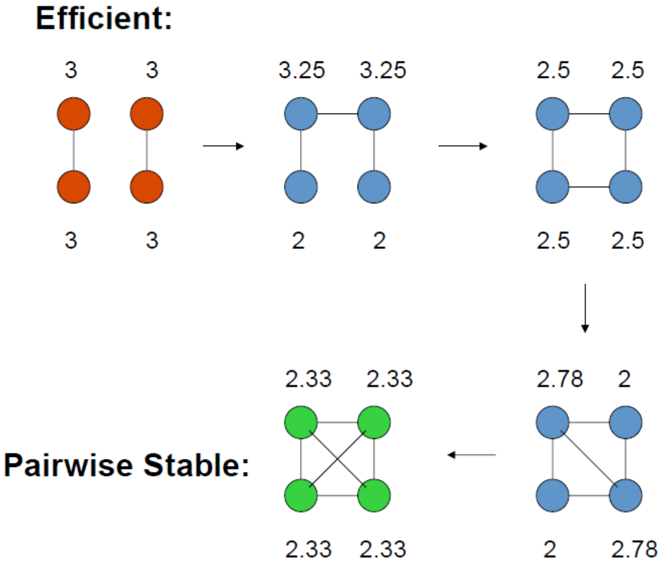
Agents obtain value from research collaborations:

- A value for each relationship: depends on the time invested in the relation.
- An interaction term: the product of the time spent.

$$u_i(g) = \sum_{j:ij \in g} [1/d_i + 1/d_j + 1/(d_i d_j)]$$

$$u_i(g) = 1 + \sum_{j:ij \in g} [1/d_j + 1/(d_i d_j)]$$

The co-author model



Results

No direct costs to links.

- **Efficient networks**: pairs
- **Pairwise stable networks**: completely connected components

Again a tension between stability and efficiency.

Thank you for your attention