Applied economic networks - V

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Today's class

- Network games
- Diego's presentation: *Altruism in networks*

What are network games?

Network

What are network games?

Network $\Longrightarrow$ Behavior

What are network games?

Network $\Longrightarrow$ Behavior $\Longrightarrow$ Payoff

What are network games?

Network$\Longrightarrow$ Behavior $\Longrightarrow$ Payoff $\Longrightarrow$ Network?

What are network games?

- Why not just a game?

What are network games?

- Why not just a game?
- We can see network games as a generalization of traditional game theory. The structure of the society matter.
- There is particular attention to the network, and comparative statics mostly focus on that.

Networks are everywhere

- Coordination problems (complementarities)
- external effects (interaction between education and connectedness)
- imperfect information (learning through the network)
- conformism
- social identity (stereotypes)

A simple example

Players choose an action $x_i \in \{0, 1\}$.

Payoff depend on:

- how many neighbors choose each action
- the number of neighbors

We can think of this game as technology adoption.

Example 1

An agent is willing to take action 1 iff at least two other neighbors do.

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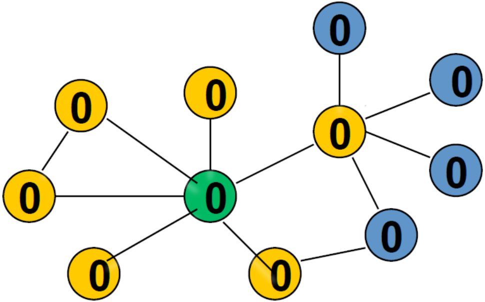

Figure: Source: these figures are taken from M. O. Jackson class material.

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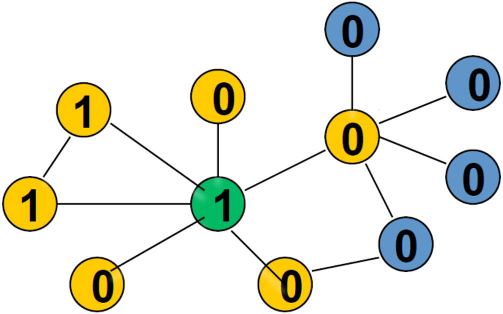

An extension

Instead of two neighbors, now an agent chooses action 1 iff at least k neighbors do.

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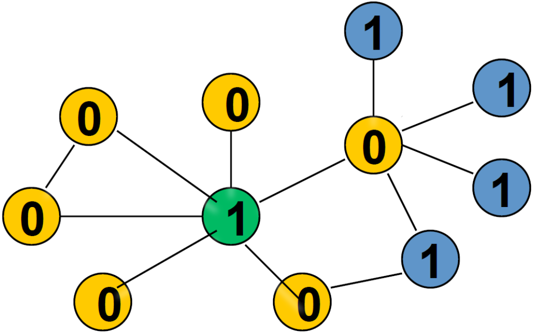
Payoff are:

- i chooses 0: $u_{d_i}(0, m_{N_i}) = 0$
- i chooses 1: $u_{d_i}(0, m_{N_i}) = -k + m_{N_i}$

Best shot public good game

In this variation, an agent chooses action 1 iff no other neighbors do so.



Best shot - payoff

- Payoff when choosing 0:

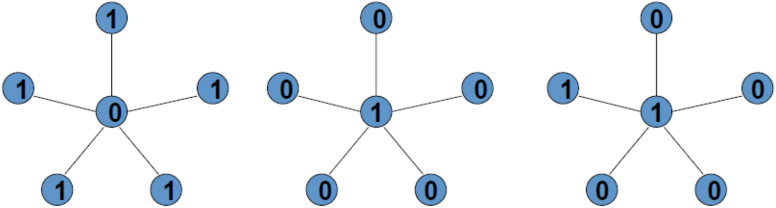
$$\begin{aligned} u_i(0, m_{N_i}) &= 1 \quad \text{if} \quad m_{N_i} > 0 \\ &= 0 \quad \text{if} \quad m_{N_i} = 0 \end{aligned}$$

- Payoff when choosing 1:

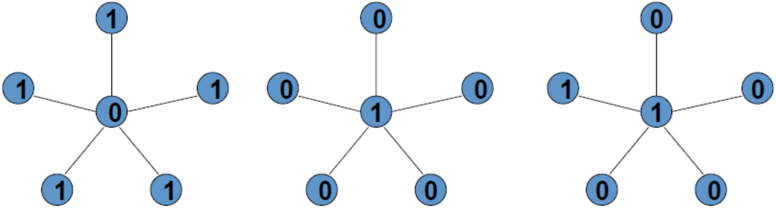
$$u_i(1, m_{N_i}) = 1 - c \tag{1}$$

for some $c > 0$.

Equilibrium in a star



Equilibrium in a star



- First network is an equilibrium
- Second network is an equilibrium
- Third network is NOT an equilibrium

Lesson from the example

We have seen (briefly) two similar but different games.

- Example 1: the choice to take an action by my friends increases the relative payoff to taking that action.
- Example 2: the choice to take an action by my friends decreases the relative payoff to taking that action.

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Strategic complements and strategic substitutes

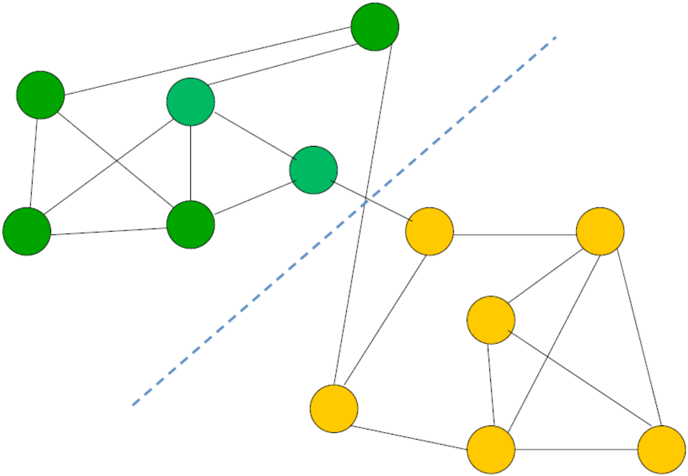
- In example 1 we have **strategic complements**;
- In example 2 we have **strategic substitutes**.

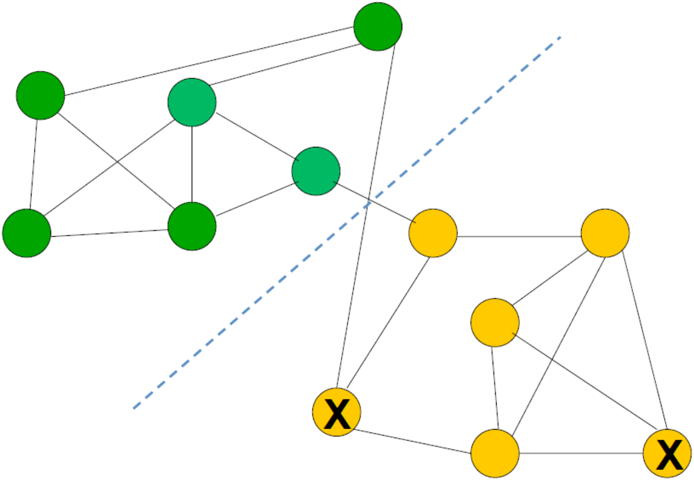
A labor market model

- The value of being in the labor market increases if friends are in the labor force
- If some friends drop out, I drop out too

The basic model can be extended in several ways:

- Heterogeneity in thresholds
- Different initial conditions
- **Homophily**

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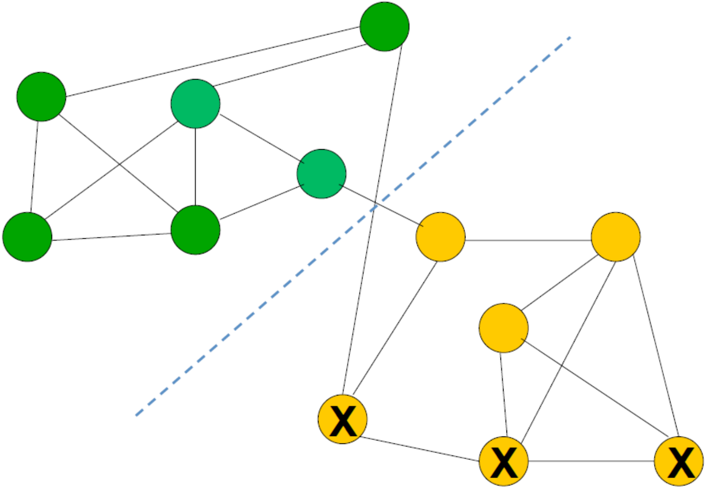
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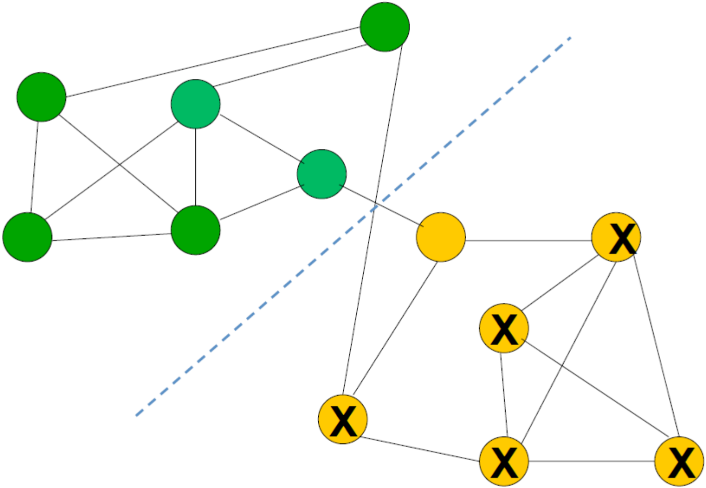
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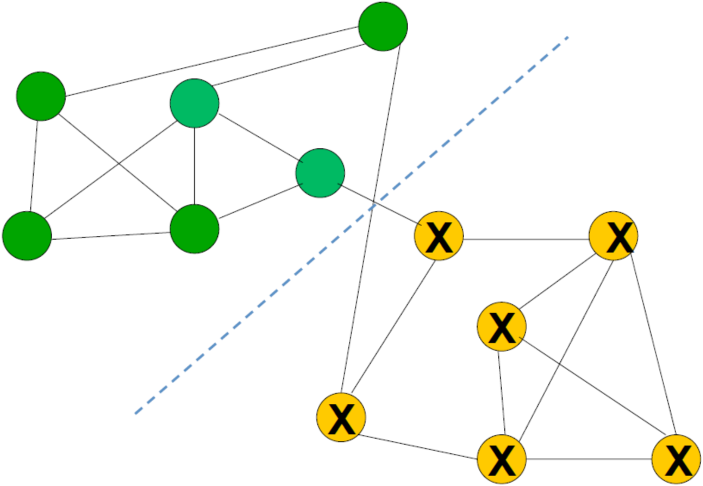
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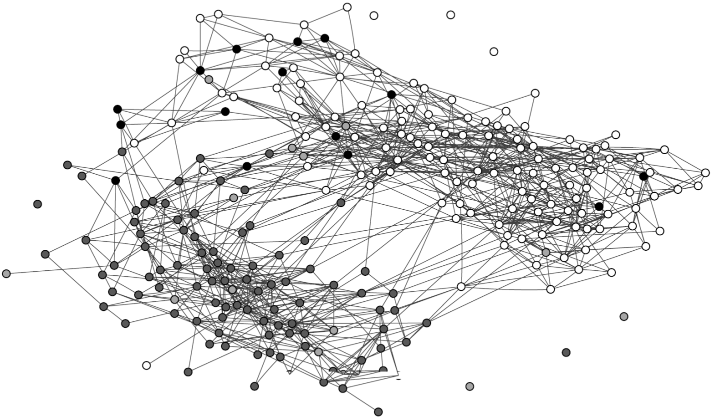
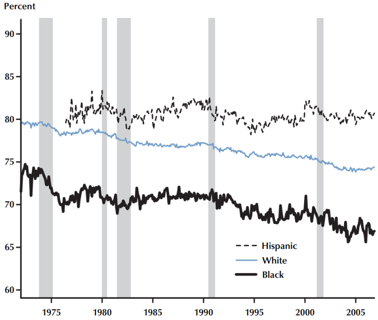
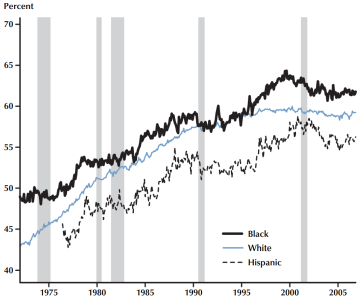


Figure: Source: *Currarini, Jackson and Pin 2009*



(a) Men



(b) Women

Figure: Source: *DiCecio et al. 2008*

A more interesting game

- Each player $i \in N = \{1, 2, \ldots, n\}$ simultaneously selects an action $a_i \geq 0$.
- Payoff depend on everyone's action $u_i(a_1, a_2, \ldots, a_n)$.

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Best response

$$BR_i(\mathbf{a}_i) = \alpha \sum_j w_{ij} a_j + b_i$$

where $\alpha > 0$.

What if $w_{ij} = \frac{1}{d_i}$ for all j such that $g_{ij} = 1$?

Why this best response?

- Can be shown that it corresponds to the linear-in-means model (peer effects)
- In a dynamic setting this is the DeGroot updating rule (naive learning – in two lessons)
- In general has deep interpretation
- It is linear..

What payoff?

The best response described before can be derived from two different payoff functions:

- $$u_i(\mathbf{a}) = -\frac{a_i^2}{2} + \alpha \sum_j w_{ij} a_i a_j + b_i a_i$$
- By setting $b_i = (1-\alpha)y_i$:

$$u_i(\mathbf{a}) = -\alpha \sum_j (a_i - a_j)^2 - (1-\alpha)(a_i - y_i)^2$$

Is this a game of strategic complements or substitutes?

Solving the game

Let us keep our assumption that $w_{ij} = 1/d_i$ for all $g_{ij} = 1$.

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- $(\mathbf{I} - \alpha \mathbf{W})\mathbf{a} = \mathbf{b}$
- We need to invert the matrix $(\mathbf{I} - \alpha \mathbf{W})$. What conditions do we need?

Solving the game - contd

- All rows of $\mathbf{W}$ sum up to 1 (row stochastic matrix).
- $\alpha \in (0, 1)$.

Result

The game has a unique Nash equilibrium given by:

$$\mathbf{a}^* = (\mathbf{I} - \alpha\mathbf{W})^{-1}\mathbf{b}$$

Moreover, $(\mathbf{I} - \alpha\mathbf{W})$ is a non-negative matrix.

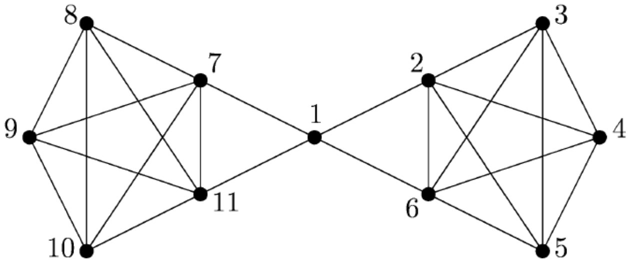
Further result

Nash equilibrium and network structure

The actions in equilibrium are directly proportional to the Bonacich centrality.

- This model has first been studied by *Ballester, Calvo-armengol and Zenou, ECMA, 2006*.

The key player

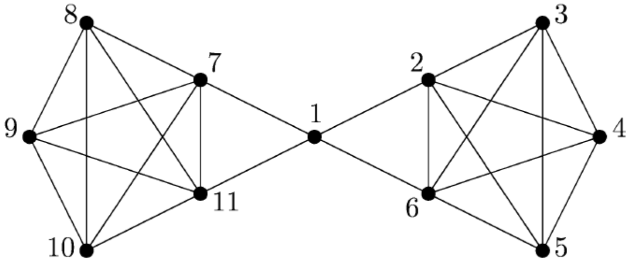


Who is the key player?

- Player 1 connects the network
- Removing player 2 we disrupt the highest number of links

Who has the highest impact on direct and indirect effects?

The key player



Who is the key player? Result from BCZ 2006

- If α is low enough player 2 is the key player
- If α is large enough, player 1 is the key player $\Longrightarrow$ Indirect effects matter more

Thank you for your attention