Applied economic networks - VII

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Today's class

- Learning in networks
- Philippine's presentation: *How homophily affects the speed of learning and best-response dynamics*
- Philemon's presentation: *Homophily and the persistence of disagreement*

Learning

- Individuals learn from actions of others,
- Individuals learn from talking with others,
- and they also process this information: inference on indirect information

Why is it different from diffusion?

We keep track of the information, so it's not just an issue of being infected or not.

Main questions

- Is dispersed information aggregated efficiently?
- Whose opinions or experiences are particularly influential?
- Will choices of individuals be the same or differ? How do the networks play a role in this?

Two main approaches

1. Individuals are rational: they observe all previous actions and a private signal.
    - Others' choices carry information about their signals,
    - **information is an externality**,
    - individuals may end up ignoring private signals and copy the crowd $\Rightarrow$ **herding**

Two main approaches

1. Individuals are rational: they observe all previous actions and a private signal.
   - Others' choices carry information about their signals,
   - **information is an externality**,
   - individuals may end up ignoring private signals and copy the crowd $\Rightarrow$ **herding**
2. Individuals have bounded rationality
   - Agents start with an exogenous opinion,
   - the opinion updates over time according to an updating rule.
   - Since it is a mechanical model, we can call it *naive learning*.

A simple Bayesian model of learning - Bala & Goyal 1998

- n players in an undirected network g
- Each period agents choose action A or B
- A gives utility 1 for sure, B gives 2 with probability p and 0 with probability $1 - p$

Learning

- At each point in time agents get payoff depending on their action
- Agents observe neighbors choices - the network plays a role
- The problem is to maximize the discounted stream of payoff
- p is unknown

updating beliefs

- Since agents do not know p, they make inference based on the history of choices and payoff
- $\mu_i(h_{i,t})$ is the inferred probability for an agent i that B pays 2, given a history of observations

Result

Proposition

If $p \neq 1/2$, with probability 1 there is a finite time t such that all agents play the same action from that time onward.

Sketch of the proof

Suppose the contrary..

- Some agent plays B infinitely often, and his belief on p converges to the true p.
- If so, the belief converges to $p > 1/2$, or otherwise she would have stopped playing B.
- All of her neighbors see B being played infinitely often..
- Neighbors of the first agent must play B, and after some time also the neighbors of the neighbors $\Rightarrow$ everyone plays B.

Conclusions

The network plays little role in the end.

Agents play B if $p > 1/2$, A otherwise.

Correct aggregation may fail, but under some very strict conditions:

- The complete network,
- The network where every agent has at most 1 neighbor.

Repeated linear updating

What if agents are not rational?

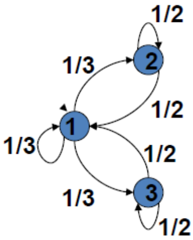
- Agents are born with some opinions.
- The network T is weighted (row-stochastic) and directed.
- The weights (and the network) do not adjust over time (different from Bayesian).

The updating rule

In every period t an agent's opinion is given by:

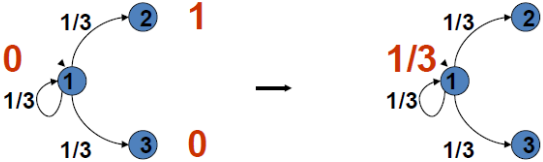
$$b_i(t) = \sum_j T_{ij} b_j(t-1)$$

An example



$$T = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

An example



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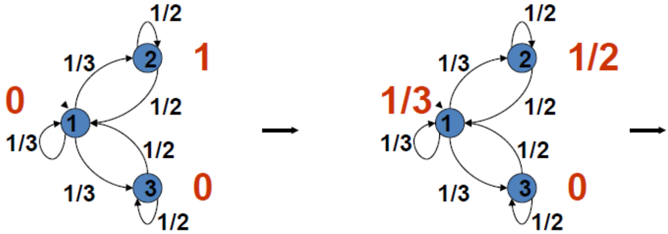

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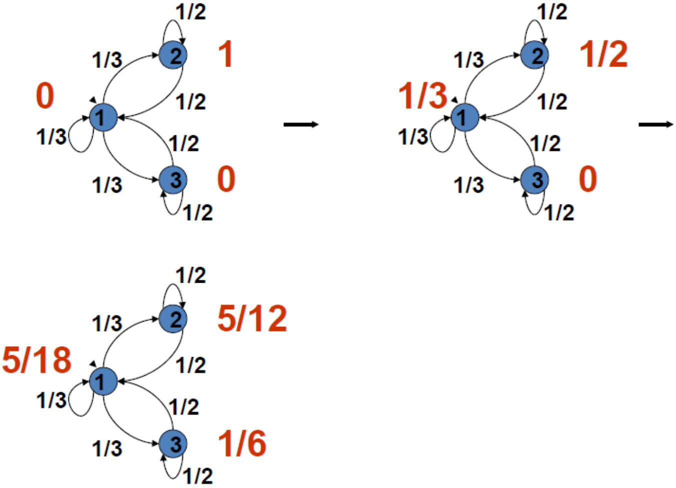
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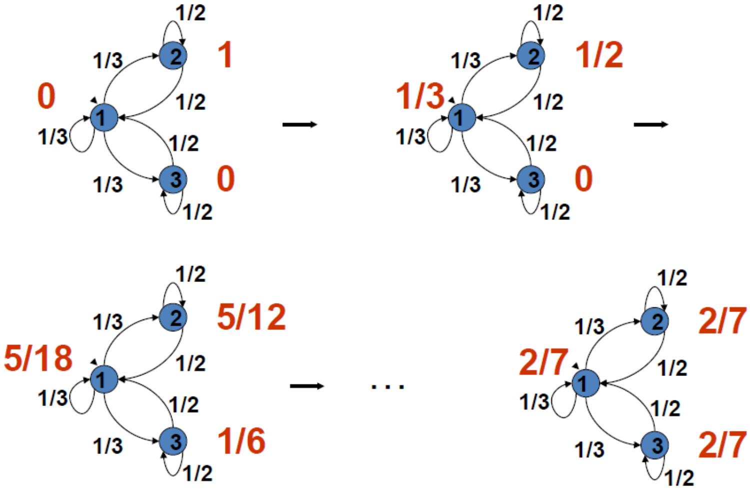
An example



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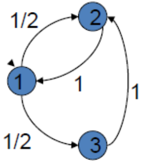
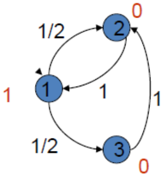
What can we say about convergence?

An example of convergence



$$T= \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

An example of convergence



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$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

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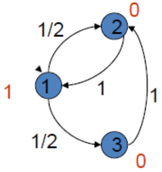

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$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

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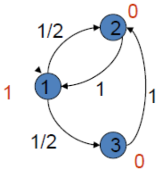
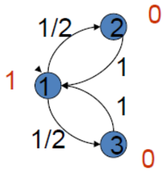


$$T = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3/4 \\ 1/2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/4 \\ 3/4 \\ 1/2 \end{pmatrix} \cdots \rightarrow \begin{pmatrix} 2/5 \\ 2/5 \\ 2/5 \end{pmatrix}$$

An example of not-convergence



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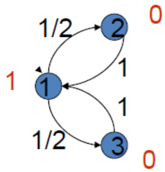

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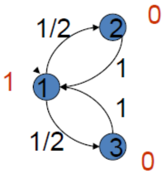

$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

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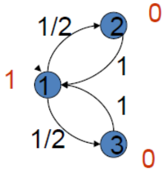

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Golub & Jackson 2010

Result

Suppose T is strongly connected (there exists a path between any two agents): T is convergent if and only if it is **aperiodic**.

Golub & Jackson 2010

Result

Suppose T is strongly connected (there exists a path between any two agents): T is convergent if and only if it is **aperiodic**.

Definition - convergence

Everyone's opinion is the same and does not change over time. $b_i(t') = b$ for all i and for all t'.

Definition - Aperiodicity

Greatest common divisor of cycle lengths is 1.

Proof of the result

We need to show that if the network is strongly connected, stochastic and aperiodic, then there is convergence.
We start with an old result:

Perron-Frobenius theorem - an extrapolation

T is primitive iff $T_{ij}^t > 0$ for all ij and for all $t > t'$, for some t'.

If so there exists a positive eigenvalue which is strictly greater in absolute value than all the other eigenvalues. Here we use stochastic matrix.

Proof of the result

Perkins (1961)

If T is strongly connected and stochastic then it is aperiodic iff it is primitive.

So, if T is stochastic, then it is primitive. If it is primitive it is aperiodic. The last piece:

Result

If T is strongly connected and primitive then :

$$\lim_t T^t = 1, 1, \ldots, 1^T s \qquad (1)$$

where s is the unique eigenvector associated with eigenvalue 1.

When is the information accurate?

Assume there is some uncertainty:

- A true state of the world μ.
- Agents see $b_i(0) = \mu + \varepsilon_i$
- If agents are able to pool the information, they have an accurate estimate of μ.

The wisdom of the crowds

Result (informal)

The crowds are wise if there are no agents "too" influential.

So wisdom depends on the social structure!

- No prominent agents (e.g. royal families)
- Reciprocal trust: attention is reciprocated (row and column stochastic matrix)

Related questions

- How quickly a society reaches consensus? How does the *speed of learning* relate to the network structure?
- Under which conditions consensus fail to exist?

Thank you for your attention